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Numerical Solution of A Zakat-Based Wealth Redistribution Model using The Explicit Euler Method

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ABSTRACT

Zakat is an important mechanism of wealth redistribution in the Islamic economic system. Mathematical modelling provides a quantitative framework for studying the dynamics associated with the transfer of wealth from eligible wealth holders to recipients. In this study, an existing continuous Zakat-based wealth redistribution model formulated by Pakdemirli is investigated numerically using the Explicit Euler method. The continuous model describes the wealth dynamics of a Zakat-paying rich group and a Zakat-receiving poor group through a system of ordinary differential equations. Its corresponding discrete formulation is derived using a uniform time-step discretization and is verified against the available analytical solution. Numerical accuracy and convergence are examined through step-size refinement, while the stability condition of the Explicit Euler scheme is analyzed for the wealth equation governing the rich group. The sensitivity of the parameters is investigated with respect to the Zakat ratio, the earnings ratio, and the constant earning parameter. In addition, the cumulative Zakat transfer is quantified over a prescribed time interval. A hypothetical numerical scenario is then used to illustrate the behavior of the discrete model and the cumulative redistribution represented by it. The study provides a systematic numerical formulation and analysis of an existing continuous Zakat-based wealth redistribution model.

Introduction

Zakat is an important institution of the Islamic economic system and serves as a mechanism for wealth redistribution and support for needy members of society. Its potential role in reducing economic inequality and alleviating poverty has been investigated in empirical research. Jehle (Jehle 1994) examined the relationship between Zakat and income inequality in Pakistan and reported a reduction in measured inequality associated with Zakat. Similarly, Kasri (Kasri 2016) investigated the

effectiveness of Zakah targeting in alleviating poverty in Indonesia. More recent research has also examined the relationship between Zakat distribution and poverty-related outcomes (Choiriyah et al. 2020). Reviews of the literature further demonstrate the continuing academic interest in Zakat and poverty alleviation (Hasibuan et al. 2024).

Mathematical modelling provides a quantitative approach for studying the economic dynamics associated with Zakat. Subhan (Subhan 2018) developed a mathematical model of economic population dynamics under optimal Zakat management, while Subhan and Sriningsih (Subhan and Sriningsih 2026) formulated a differential-equation model incorporating Zakat and low-tax policies. These studies demonstrate how mathematical models can represent and investigate Zakat-related economic processes.

Among these approaches, Pakdemirli (Pakdemirli 2023) proposed a continuous mathematical model describing wealth transfer from a rich group subject to Zakat to a poor group receiving Zakat. Let $r(t)$ denote the total wealth of the rich group and $p(t)$ the total wealth of the poor group. Before the poor group reaches the specified richness limit g_0, the model is given by

$$\frac{dr}{dt} = (\beta - \alpha)r, \qquad \frac{dp}{dt} = \alpha r + \mu, \tag{1}$$

with initial conditions

$$r(0) = r_0, \qquad p(0) = p_0. \tag{2}$$

Here, α represents the Zakat ratio, β the earnings ratio of the rich group, and μ the constant earning rate of the poor group. The term αr represents the rate of Zakat transfer. The model also specifies a richness limit for the poor group, after which the transfer dynamics are modified (Pakdemirli 2023).

Although the continuous model admits analytical solutions, its numerical treatment provides a framework for examining approximation error, convergence, stability, and the effects of model parameters under discrete time steps. The Explicit Euler method is a first-order numerical method for approximating solutions of ordinary differential equations. Its convergence and stability properties are established in the numerical-analysis literature (Butcher 2016; Hairer et al. 2008). Applying this method to the adopted Zakat-based model allows the continuous formulation to be expressed as a computational recurrence system and provides a basis for assessing the accuracy of the resulting numerical solutions. The current continuous Zakat-Based Wealth Redistribution model is developed and analysed in this research with the help of the Explicit Euler discretization. Absolute errors and Relative errors have been performed between the numerical solution and the analytical solution to demonstrate convergence of the numerical solution towards the analytical solution, and systematic refinement of the time-step is used to demonstrate convergence. The stability properties of the numerical scheme are also examined for some range of the parameter. Moreover, sensitivity analysis is carried out with respect to the Zakat ratio α and the earnings ratio β, and the earning rate of the poor group μ. A hypothetical numerical case is also included that shows the computational framework and the cumulative transfer of Zakat is scaled over the active redistribution interval.

This is a numerical and computational study. It does not introduce a new economic model or Zakat model; rather, it is a numerical and systematic analysis of an existing continuous model with respect to the accuracy of the solutions, time-step convergence, stability, and sensitivity with respect to the parameters and cumulative transfer.

Mathematical Model

Continuous Wealth Redistribution Model

Consider two representative groups: a rich group whose wealth is subject to Zakat and a poor group that receives the corresponding transfer. Let

$$r(t)$$

denote the total wealth of the rich group and

$$p(t)$$

denote the total wealth of the poor group.

Following Pakdemirli (Pakdemirli 2023), the continuous model before the poor group reaches the richness limit g_0 is

$$\frac{dr}{dt} = (\beta - \alpha)r, \qquad \frac{dp}{dt} = \alpha r + \mu. \tag{3}$$

The parameters are defined as follows:

- $r(t)$: $wealth\ of\ the\ rich\ group$;
- $p(t)$: $wealth\ of\ the\ poor\ group$;
- α: $Zakat\ ratio$;
- β: $earnings\ ratio\ of\ the\ rich\ group$;
- μ: $constant\ earning\ rate\ of\ the\ poor\ group$;
- r_0: $initial\ wealth\ of\ the\ rich\ group$;
- p_0: $initial\ wealth\ of\ the\ poor\ group$;
- g_0: $richness\ limit\ for\ the\ poor\ group$.

The initial conditions are

$$r(0) = r_0, \qquad p(0) = p_0. \tag{4}$$

The model assumes that the earnings of the rich group are proportional to its wealth, whereas the Zakat payment is also proportional to its wealth. The poor group receives the Zakat transfer αr and has an additional constant earning rate μ. These assumptions follow the continuous wealth redistribution model proposed by Pakdemirli (Pakdemirli 2023).

Analytical Solution

The first equation of (3) is separable:

$$\frac{1}{r}\frac{dr}{dt} = \beta - \alpha. \tag{5}$$

Integrating and applying $r(0) = r_0$ gives

$$r(t) = r_0 e^{(\beta-\alpha)t}. \tag{6}$$

Substituting (6) into the equation for $p(t)$ gives

$$\frac{dp}{dt} = \alpha r_0 e^{(\beta-\alpha)t} + \mu. \tag{7}$$

For $\beta \neq \alpha$, integration gives

$$p(t) = p_0 + \frac{\alpha r_0}{\beta - \alpha}\left[e^{(\beta-\alpha)t} - 1\right] + \mu t. \tag{8}$$

Therefore, before the richness limit is reached, the analytical solution is

$$\boxed{r(t) = r_0 e^{(\beta-\alpha)t}} \tag{9}$$

and

$$\boxed{p(t) = p_0 + \frac{\alpha r_0}{\beta - \alpha}\left[e^{(\beta-\alpha)t} - 1\right] + \mu t}. \tag{10}$$

For the special case $\beta = \alpha$,

$$r(t) = r_0, \tag{11}$$

and

$$p(t) = p_0 + (\alpha r_0 + \mu)t. \tag{12}$$

Let the interval $[0, T]$ be divided into N uniform subintervals. The time-step size is

$$h = \frac{T}{N}, \tag{13}$$

with grid points

$$t_n = nh, \qquad n = 0,1, \dots, N. \tag{14}$$

For a general initial-value problem, the Explicit Euler method is a first-order numerical method for ordinary differential equations (Butcher 2016; Hairer et al. 2008).

$$y'(t) = f\big(t, y(t)\big), \tag{15}$$

the Explicit Euler method is given by

$$y_{n+1} = y_n + hf(t_n, y_n). \tag{16}$$

Applying this method to the rich-group equation gives

$$r_{n+1} = r_n + h(\beta - \alpha)r_n, \tag{17}$$

or

$$\boxed{r_{n+1} = [1 + h(\beta - \alpha)]r_n.} \tag{18}$$

Similarly, the poor-group equation gives

$$\boxed{p_{n+1} = p_n + h(\alpha r_n + \mu).} \tag{19}$$

Hence, the Explicit Euler approximation to the continuous model is

$$\boxed{\begin{aligned} r_{n+1} &= [1 + h(\beta - \alpha)]r_n, \\ p_{n+1} &= p_n + h(\alpha r_n + \mu). \end{aligned}} \tag{20}$$

with

$$r_0 = r(0), \qquad p_0 = p(0). \tag{21}$$

The numerical scheme is applied only while the poor-group wealth satisfies

$$p_n < g_0. \tag{22}$$

If the numerical solution reaches or exceeds g_0, the corresponding threshold time must be identified and the post-threshold formulation must be treated separately, consistently with the original model.

Numerical Accuracy and Convergence

Absolute Error

Let $r(t_n)$ and $p(t_n)$ denote the analytical solutions and r_n and p_n the corresponding Explicit Euler approximations.

The absolute errors are defined by

$$E_r(n) = |r(t_n) - r_n|, \tag{23}$$

and

$$E_p(n) = |p(t_n) - p_n|. \tag{24}$$

The maximum absolute errors over the computational interval are

$$E_r^{\max} = \max_{0 \le n \le N} E_r(n), \tag{25}$$

and

$$E_p^{\max} = \max_{0 \le n \le N} E_p(n). \tag{26}$$

Relative and Percentage Error

When the analytical solution is nonzero, the relative errors may be computed as

$$RE_r(n) = \frac{|r(t_n) - r_n|}{|r(t_n)|}, \tag{27}$$

and

$$RE_p(n) = \frac{|p(t_n) - p_n|}{|p(t_n)|}. \tag{28}$$

The percentage errors are

$$PE_r(n) = 100RE_r(n), \tag{29}$$

and

$$PE_p(n) = 100RE_p(n). \tag{30}$$

Convergence

The convergence of the Explicit Euler approximation is investigated by reducing the time-step size and observing the corresponding reduction in numerical error.

For two consecutive step sizes h and $h/2$, the observed order of convergence may be estimated by

$$q = \frac{\log\left(E_h \big/ E_{\frac{h}{2}}\right)}{\log(2)}, \tag{31}$$

where E_h and $E_{h/2}$ denote the selected error measure for the two computations.

For sufficiently smooth solutions, the Explicit Euler method has global order one under the standard assumptions for convergence of numerical initial-value methods (Butcher 2016; Hairer et al. 2008). Consequently, the global error is expected to decrease approximately in proportion to the time-step size when the computation is in the asymptotic convergence regime.

Stability Analysis

The absolute stability of the Explicit Euler method is examined using the standard linear test equation (Butcher 2016; Hairer et al. 2008)

$$y' = \lambda y. \tag{32}$$

Its Explicit Euler discretization is

$$y_{n+1} = (1 + h\lambda)y_n. \tag{33}$$

For the rich-group equation,

$$\lambda = \beta - \alpha, \tag{34}$$

and therefore the amplification factor is

$$G = 1 + h(\beta - \alpha). \tag{35}$$

The absolute stability condition is

$$\boxed{|1 + h(\beta - \alpha)| < 1.} \tag{36}$$

When $\beta - \alpha < 0$, this gives

$$0 < h < \frac{2}{\alpha - \beta}. \tag{37}$$

When $\beta - \alpha \geq 0$, the continuous rich-group solution is non-decaying and the standard absolute-stability restriction for a decaying test equation does not apply in the same way. In this case, the numerical behavior is examined by comparison with the exact solution and by monitoring the growth of numerical errors.

The poor-group equation is driven by the rich-group wealth trajectory and contains the same coefficient $\beta - \alpha$ through its dependence on $r(t)$. The amplification factor therefore describes the homogeneous rich-group component of the coupled system. The numerical behaviour of both wealth variables is additionally assessed by comparing their computed values with the corresponding analytical solutions.

Parameter Sensitivity Analysis

The sensitivity of the wealth trajectories to the model parameters is investigated by varying one parameter at a time while keeping the remaining parameters fixed.

The parameters considered are

$$\alpha, \qquad \beta, \qquad \mu. \tag{38}$$

The parameter α controls the Zakat transfer rate $\alpha r(t)$, while β controls the proportional earnings of the rich group. The parameter μ represents the constant earning rate of the poor group.

For each parameter, the effects on

1. the rich-group wealth $r(t)$,
2. the poor-group wealth $p(t)$,
3. the instantaneous Zakat transfer $\alpha r(t)$, and
4. the cumulative Zakat transfer

   are examined numerically.

Cumulative Zakat Transfer

Within the model, the instantaneous Zakat transfer from the rich group to the poor group is

$$Z(t) = \alpha r(t). \tag{39}$$

Therefore, the cumulative Zakat transferred over the interval $[0, T]$ is defined by

$$\boxed{Z_{\text{cum}}(T) = \int_0^T \alpha\, r(t)\, dt.} \tag{40}$$

This quantity is evaluated only over the interval during which the poor group remains eligible to receive the transfer under the assumptions of the model.

If $T \leq t_1$, where t_1 denotes the time at which $p(t_1) = g_0$, then using (6) gives

$$Z_{\text{cum}}(T) = \alpha r_0 \int_0^T e^{(\beta-\alpha)t}\, dt. \tag{41}$$

For $\beta \neq \alpha$,

$$\boxed{Z_{\text{cum}}(T) = \frac{\alpha r_0}{\beta - \alpha}\left[e^{(\beta-\alpha)T} - 1\right].} \tag{42}$$

For $\beta = \alpha$,

$$\boxed{Z_{\text{cum}}(T) = \alpha r_0 T.} \tag{43}$$

A corresponding discrete approximation can be obtained using the computed Euler values,

$$Z_{\text{cum}}^{(h)} = h \sum_{n=0}^{N-1} \alpha\, r_n. \tag{44}$$

The analytical and discrete cumulative transfers are compared to assess the numerical accuracy of this quantity.

Hypothetical Numerical Scenario

A hypothetical scenario, based on the first numerical case in Pakdemirli (Pakdemirli 2023) is considered to illustrate the numerical implementation of the model. The initial amounts of wealth are $r_0 = 100$ and $p_0 = 20$. The ratio of wealth for the Zakat is $\alpha = 0.025$. The ratio of wealth for the rich group is $\beta = 0.05$. Zakat is distributed among the poor group at $\mu = 1$. The value of go is 50.

The numerical solution is obtained for $0 \le t \le 5$ with the baseline time-step size $\Delta t = 0.1$ and by the explicit Euler method. The numerical accuracy and convergence is examined by calculating with time-step size of $\Delta t = 0.025$ and $\Delta t = 0.05$. The present numerical study is performed for these time-steps and for a computational time interval of five years.

The calculated wealth functions are compared to the analytical solutions. Absolute errors are computed to assess the numerical accuracy and the observed convergence order is explored by refining the time-step. The numerical approximation and the analytical expression of the cumulative Zakat transfer are both used for the same interval. Finally, the effect of the parameters on the outcomes is tested by changing the value of the number of Zakat ratio, the number of Zakat earnings ratio of the rich group and the income constant of the poor group while keeping other parameters constant.

Because the baseline five-year interval for the poor group is below the threshold $g_0 = 50$ we are using the pre-threshold equations in this experiment. The numerical results are not intended to be estimates of real Zakat collections or distributions of wealth, but merely as an illustration of the mathematical model.

Numerical Results

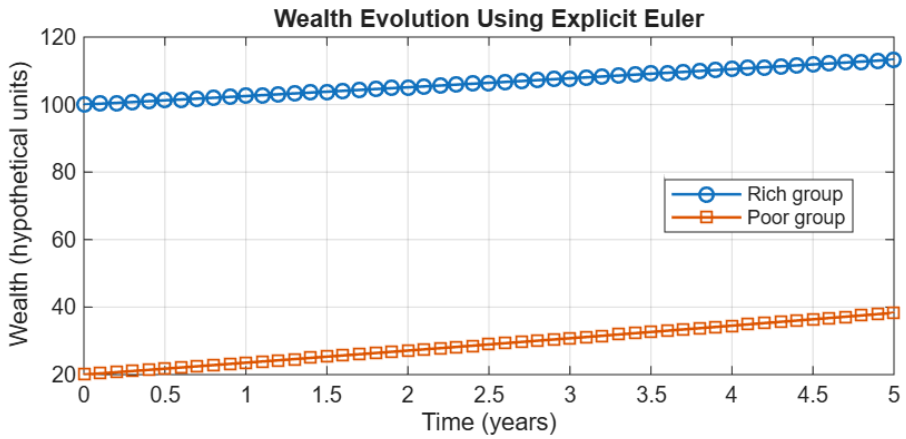
Table 1 presents the numerical results obtained Use the Explicit Euler method to numerically solve the initial value problem above, taking 0.1 for the time-step size, Δt. The numerical approximations are checked with the analytical solution The solutions are given at integer values of t between 0 and 5.

Comparison of Explicit Euler and analytical solutions for the baseline scenario ($\Delta t = 0.1$).

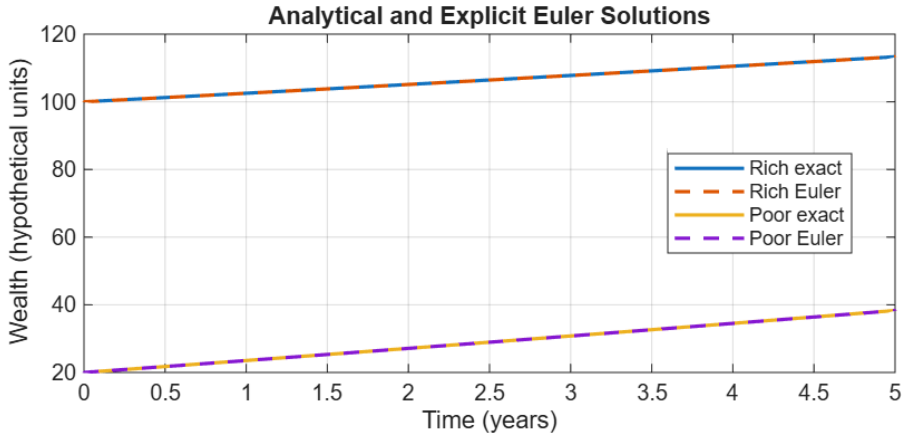
t	r_{Euler}	r_{Exact}	E_r	p_{Euler}	p_{Exact}	E_p	Z_{cum}
0	100.0000	100.0000	0.0000	20.0000	20.0000	0.0000	0.0000
1	102.5300	102.5300	0.0032	23.5280	23.5320	0.0032	2.5283
2	105.1200	105.1300	0.0066	27.1210	27.1270	0.0066	5.1206
3	107.7800	107.7900	0.0101	30.7780	30.7880	0.0101	7.7783
4	110.5000	110.5200	0.0138	34.5030	34.5170	0.0138	10.5030
5	113.2972	113.3148	0.0177	38.2972	38.3148	0.0177	13.2972

The maximum absolute error in the rich group's wealth is $1.767461216 \times 10^{-2}$, while the maximum absolute error in the poor group's wealth is also $1.767461216 \times 10^{-2}$. At $t = 5$, the Explicit Euler approximation gives $r(5) \approx 113.2972$ and $p(5) \approx 38.2972$, whereas the analytical solutions give $r(5) \approx 113.3148$ and $p(5) \approx 38.3148$, respectively.

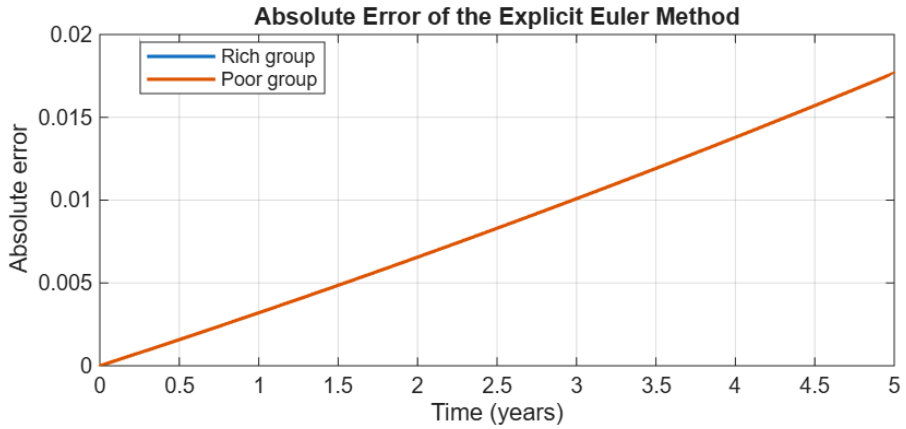
The cumulative Zakat transfer computed using the Explicit Euler method is approximately 13.2972, compared with the analytical value of 13.3148. The numerical results show a small difference between the Euler approximations and the analytical solutions over the selected time interval. Furthermore, the poor group's wealth remains below the threshold $g_0 = 50$ throughout the five-year interval. Therefore, the pre-threshold equations remain applicable throughout this experiment.



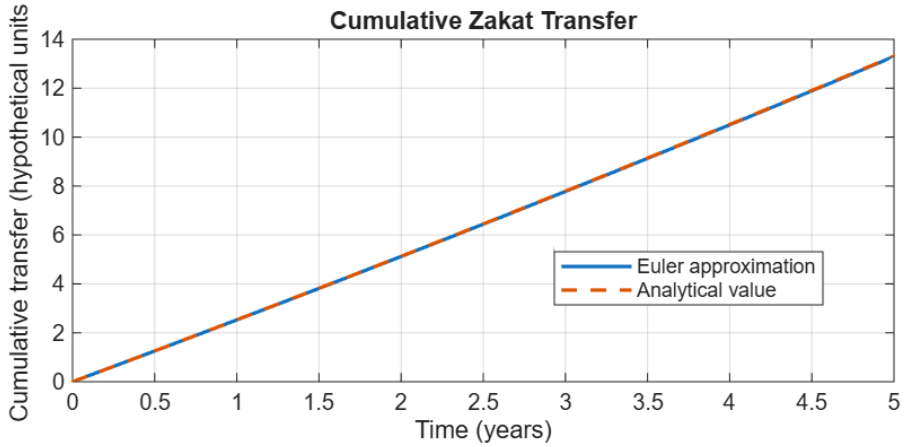
Wealth trajectories of the rich and poor groups over the five-year interval using the Explicit Euler method.



Comparison of the Explicit Euler approximations with the corresponding analytical solutions.



Absolute errors between the Explicit Euler approximations and the analytical solutions.



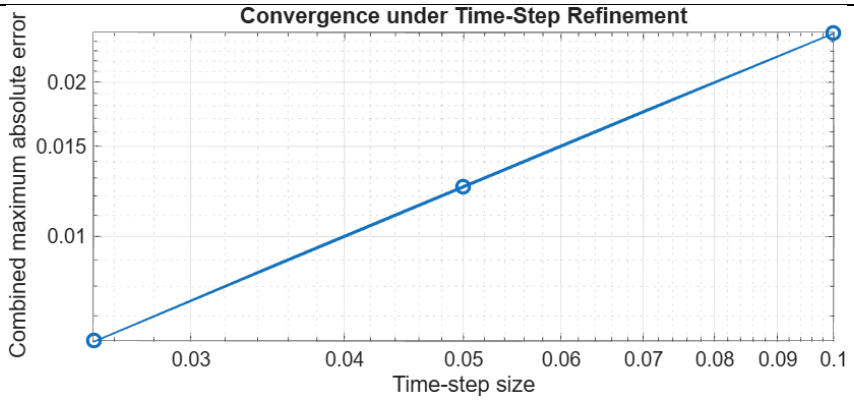
Comparison of cumulative Zakat transfer calculated using the Explicit Euler method and the analytical expression.

Convergence Analysis

To investigate the convergence of Explicit Euler method, The computation is repeated for three time-step sizes: $\Delta t = 0.1$, $\Delta t = 0.05$, and $\Delta t = 0.025$. The largest absolute errors and rates of convergence observed are shown. The results of the convergence are reported in Table 2.

Convergence results under time-step refinement.

Δt	Maximum Rich Error	Maximum Poor Error	Combined Error	Observed Order
0.100	0.017675	0.017675	0.024996	–
0.050	0.008845	0.008845	0.012509	0.99874
0.025	0.0044244	0.0044244	0.0062571	0.99937



Convergence of the Explicit Euler method under time-step refinement.

The results in Table 2 show that the maximum absolute errors decrease as the time-step size is reduced. When the time-step size is halved, the combined error is approximately halved. The observed convergence orders are 0.99874 and 0.99937, which are close to the theoretical order of one for the Explicit Euler method. These results support the first-order convergence of the numerical method for the selected model and parameter values. Figure 5 illustrates this error reduction under time-step refinement.

Parameter Sensitivity Analysis

A parameter sensitivity analysis was carried out to examine the sensitivity of changes to the The ratio of the earnings of the rich-group to the earnings of the poor-group, β and Numerical results are not affected by the poor-group earnings rate μ. All of the parameters were changed one at a time, with the other parameters remaining fixed. Baseline values of were maintained. The Explicit Euler method was used. with $\Delta t = 0.1$ over the interval $T = 5$.

Sensitivity analysis with respect to the Zakat ratio α.

α	Final Poor Wealth	Cumulative Zakat
0.020	35.772	10.772
0.025	38.297	13.297
0.030	40.759	15.759

Table 3; shows that increasing α increases both final poor-group wealth and cumulative Zakat transfer for the selected parameter values.

Sensitivity analysis with respect to the rich-group earnings ratio β.

β	Final Poor Wealth	Cumulative Zakat
0.040	37.971	12.971
0.050	38.297	13.297
0.060	38.634	13.634

As shown in Table 4, increasing β produces a modest increase in final poor-group wealth and cumulative Zakat transfer in the selected numerical scenario.

Sensitivity analysis with respect to the poor-group earnings rate μ.

μ	Final Poor Wealth	Cumulative Zakat
0.5	35.797	13.297
1.0	38.297	13.297
1.5	40.797	13.297

Table 5 indicates that increasing μ increases final poor-group wealth, while cumulative Zakat remains unchanged. This occurs because μ contributes directly to the poor group's wealth equation but does not appear in the Zakat transfer term $\alpha r(t)$.

Overall, the sensitivity analysis demonstrates that the selected parameters affect the model outputs in different ways. The Zakat ratio α directly influences the transfer rate, whereas μ affects poor-group wealth independently of the cumulative Zakat calculation. These findings apply to the hypothetical parameter values considered in this study.

Theoretical Stability Analysis

The Explicit Euler method was investigated with regard to its theoretical stability. In this case, the equation is known as the linear rich-group equation, and is given by:

$$\frac{dr}{dt} = (\beta - \alpha)r.$$

The following parameters were used for the stability test: $\alpha = 0.025$ and $\beta = 0.015$. Therefore,

$$\lambda = \beta - \alpha = -0.01.$$

The Explicit Euler method amplification factor is

$$G = 1 + \Delta t\, \lambda.$$

The absolute stability condition is

$$|G| < 1.$$

Since $\lambda < 0$, this condition gives

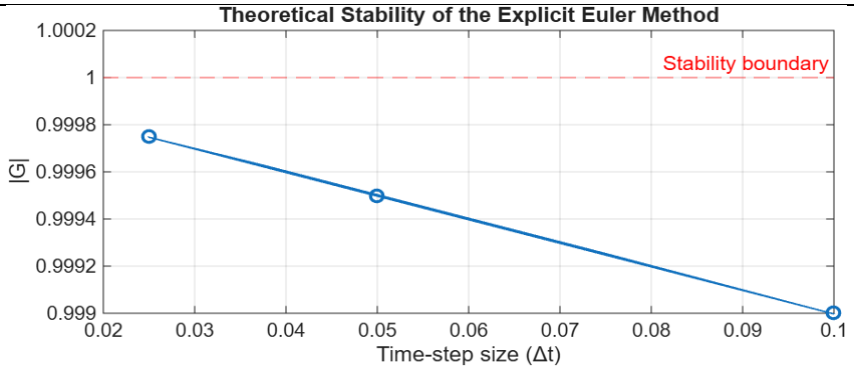
$$0 < \Delta t < \frac{-2}{\lambda} = 200.$$

The amplification factors for the following are contained in Table 6: Three different time-step sizes were chosen.

Table 6; presents the amplification factors for three selected time-step sizes.

Theoretical stability results for the Explicit Euler method.

Time Step Δt	Amplification Factor G	$\lvert G \rvert$	Absolutely Stable
0.100	0.99900	0.99900	Yes
0.050	0.99950	0.99950	Yes
0.025	0.99975	0.99975	Yes



Absolute amplification factor of the Explicit Euler method for different time-step sizes. The dashed line represents the stability boundary $|G| = 1$.

Table 6 and Figure 6 show the theoretical stability of the Explicit Euler method for the parameter values used. Because of the negative sign of $\lambda = -0.01 = \beta - \alpha$, the absolute stability condition is $0 < \Delta t < 200$. All three tested time-step sizes, $\Delta t = 0.1$, 0.05, and 0.025, satisfy this condition, with amplification factors of 0.999, 0.9995, and 0.99975, respectively. It is seen that the absolute amplification factors are less than the stability boundary, |G|<1. The results are in agreement with the ones listed above, as the Explicit Euler method is also absolutely stable for the linear mode and the parameters used here. This analysis, however, does not determine the stability of each part of the full model.

Results and Discussion

The Explicit Euler method was applied to the hypothetical Zakat-based wealth redistribution model over $0 \leq t \leq 5$, using $r_0 = 100$, $p_0 = 20$, $\alpha = 0.025$, $\beta = 0.05$, $\mu = 1$, and $h = 0.1$.

At $T = 5$, the numerical values of rich-group and poor-group wealth were approximately 113.297171 and 38.297171, respectively, compared with the analytical values of 113.314845 and 38.314845. The maximum absolute errors of both variables was around 0.017675.

The results of the convergence analysis indicated that the numerical errors became lower as the step size became smaller, and the numerical order of convergence was nearly 1, which is consistent with the theoretical convergence order of one for the Explicit Euler method.

Using sensitivity analysis, it was found that the parameters α, β and μ affect the results differently. In the base case, cumulative Zakat transfer at $T = 5$ was around 13.297171 while with the analytical value was 13.314845 leading to the relative difference of about 0.1326%.

Another stability test with $\alpha = 0.025$ and $\beta = 0.015$ met the absolute-stability requirement for the step sizes studied. This is only true for the selected parameter case and the associated linear mode. The poor group wealth was always kept below the value $g_0 = 50$ in the baseline simulation.

The overall results illustrate the use of the Explicit Euler method to investigate numerical accuracy, convergence, parameter sensitivity and stability in the model used. The parameters are hypothetical, and the results are based on numerical behavior and not actual results of Zakat in reality.

The scope and limitations of the model are described here.

In this study, we use the continuous mathematical model created by Pakdemirli (Pakdemirli 2023). The model depicts the level of wealth with two groups of aggregates, proportional earning for the rich group, constant earning for the poor group and Zakat transfer depicted by $ar(t)$ with the assumption that they are proportional to the level of wealth. It does not consider the difference between households, consumption, investment, inflation, population growth, taxation or empirical data collection of Zakat.

Thus the numerical results reflect the behaviour of the mathematical model, and not real economic results. The hypothetical scenario shows the proposed computation approach and is not an estimation of the impact of Zakat in real life in any specific society.

Conclusion

The present study numerically analyzed the existing continuous Wealth redistribution model for Zakat based on Explicit Euler method. method. This is a continuous model of the wealth dynamics of a Zakat-paying rich group and a Zakat-receiving poor group was Discretized with uniform time grid.

The resulting numerical scheme was compared with the existing A solution of the problem was obtained by analytical methods, which was used to verify the implementation of the solution and its accuracy. The idea of convergence was explored by refining the time steps, and the The Explicit Euler method was analyzed for stability condition of the controlling linear wealth equation. Parameter sensitivity analysis To explore the effect of Zakat ratio, was carried out, As opposed to the modelled wealth, earnings ratio and constant earning rate. trajectories.

The study also calculated the total amount of Zakat transferred which is equal to The time where the Zakat transfer mechanism is in effect. A hypothetical numerical case was presented showing how the discrete The framework can be applied to analyse the evolution of wealth and the redistribution of wealth. quantitatively.

The primary contribution is thus the systematic numerical treatment of an existing continuous Zakat-based wealth redistribution model. The work contains verified Explicit Euler formulation and A combination of numerical accuracy, convergence, stability, parameter Each of these analyses, within a single instrument and on one day, was carried out separately.These analyses were done individually per instrument, per day. computational framework. Future studies can use this analysis to be extended to alternative numerical schemes, time dependent parameters, empirical More elaborate models of wealth heterogeneity, or data.

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