



Mathematical Modeling of Epidemic and Population Dynamics Systems

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ABSTRACT

Epidemiics and population dynamics mathematical modeling is an important instrument of comprehending the dissemination of dreadful diseases, anticipating population changes, and assessing intervention plans. The SIR (Susceptible-Infectious-Recovered), SEIR (Susceptible-Exposed-Infectious-Recovered), and Lotka-Volterra are examples of models used to study the relationship between groups of people and the disease. These models apply the concept of differential equations to the rates of change in populations and disease compartments over time to allow the researcher and the policy makers to predict the peak of an epidemic, assess vaccination policies, and the stability of the population in most cases. Complex dynamics (nonlinearity, stochasticity, and time-dependent intervention) can be studied using mathematical models with the help of computational simulations. This paper provides an overview of the major mathematical models in epidemic and population dynamics modeling, and particularly their uses in the planning of the health of the populace, ecological research and resource management.

Introduction

Mathematical modeling has turned out to be an essential instrument of epidemiology and population dynamics, which offers an ordered way of grasping, forecasting and controlling complex biological and ecological procedures (Anderson and May, 1991). With the variables of population and disease states in mathematical equations, researchers can determine the dynamics of infectious agent spread, population size changes and interventions and their long-term effects on the system. The initial models, including the SIR model proposed by Kermack and McKendrick (1927), formed the basis of the study of the epidemic development through compartmental modeling, separate people into susceptible, infectious, and recovered individuals. Through this method, the epidemic threshold, level of herd immunity, and the basic reproduction number (R_0) can be analyzed in a quantitative way and are at the heart of the prediction of the potential of outbreaks.

Modelling Population dynamics Population dynamics modelling, commonly denoted by the LotkaVoltaire system, models species interactions, including predatorprey interactions, competition and resource limitation (Lotka, 1925; Volterra, 1926). These models can give the information about the behavior of the population growth and stability and oscillation and these can be used in the ecology, conservation biology and resources management. Combined with epidemic models, population dynamics frameworks are used to understand the effects on the disease spreads based on the size of the host population, birth and death rates, and environmental conditions (Hethcote, 2000; Brauer and Castillo-Chavez, 2012).

Mathematical models have been generalized over the past decades to adopt more complex epidemic structures. SEIR model incorporates an exposed compartment to include incubation to make it a more realistic model of diseases with a latency phase, like measles, flu, and Covid-19 (Diekmann and Heesterbeek, 2000). Predictions can subsequently be narrowed down further using age-structured models, spatial models, and network-based models, thus considering heterogeneity in contact patterns, movement, and social behavior (Keeling and Rohani, 2008; Pastor-Satorras et al., 2015). With these developments, targeted interventions, such as prioritization of vaccination, quarantine, and social distancing policies can be quantitatively evaluated and then implemented.

With more and more complex models being formulated, computational simulations have become a part of epidemic and population dynamics modeling. Differential equations, stochastic simulations and agent-based models offer the power to do analytically inexplicable scenarios (Allen, 2008; Grassly and Fraser, 2008). To give an example, the stochastic models are able to model random effects in small populations or at an early stage of an outbreak which can greatly influence the results of the epidemic as opposed to the deterministic models. On the same note, individual-level modeling can be developed using agent-based simulations, where behavior and movement and interactions in networks are included, which is critical in identifying the dynamics of epidemics in the real world.

Ecological and public health decision-making have also been supported by integration of the epidemic and population dynamics models. As an example, one can use models to predict the effects of population density, migration, and resource level on the disease, and use the results to guide policies in human and animal population (Anderson and May, 1991; Heesterbeek et al., 2015). Modelling of influenza, Ebola, COVID-19, malaria, and other infectious diseases has created a mathematical model that shows the usefulness and significance of the tools in an outbreak and understanding the stability of a population.

To sum up, mathematical modeling of epidemic and population dynamics systems provides a quantitative model of the complex biological and ecological processes. Using the compartmental models, the population growth framework and the computer simulations, researchers are able to study the spread of diseases, interventions and the population patterns more accurately. With the emergence of new diseases and the escalation of population pressures, the creation and optimization of mathematical models will keep on contributing to the central role of the population health planning, ecological management, and resource sustainability (Brauer et al., 2019; Keeling and Rohani, 2011). In subsequent parts of this paper, the theoretical basis, calculation methods and uses of such models in the modern studies shall be expounded.

Literature Review

Since the beginning of the 20th century, mathematical epidemiology and mathematical modeling of population dynamics have been major tools in the study of biological systems. Basic problems in epidemic modelling Early epidemic modelling work focused on the SIR (Susceptible-Infectious-Recovered) model which subdivides a population into epidemic modelling classes so as to describe the way disease spreads through a population over time (Kermack & McKendrick, 1927; as reviewed in Mata and Dourado, 2021). Original SIR framework and its variants such as SEIR (Susceptible-Exposed-Infectious-Recovered) offer fundamental tools regarding the dynamics of diseases such as reproduction number estimation and epidemic thresholds that are important in planning activities of the population about their health (Abdulkareem, 2025; Khair, 2024; Mata & Dourado, 2021).

Early compartmentalized epidemic frameworks were based on deterministic ordinary differential equations (ODEs) of mixing of homogeneous mixing, and fixed transmission parameters (Dadlani et al., 2020). Deterministic methods provide insight into the deterministic modeling of disease transmission, however, when the population is heterogeneous in either susceptibility or contacting. It has also been restricted, giving rise to heterogeneous epidemic models capable of taking into account the individual variation in key parameters like infectivity and susceptibility with more realistic predictions of the ultimate epidemic sizes and the outbreak curves (Novozhilov, 2012).

In later development of epidemic modeling, there were the introduction of stochastic and discrete modelling. One such model is the Reed Frost model that operates based on the chain binomial distributions to model disease dynamics at each generation step, and it includes randomness in the interaction between individuals and offers understanding of the probabilistic nature of epidemics, particularly in small populations (Reed-Frost model, n.d.; Khair, 2024). Delay differential equations which simulate incubation periods or space effects are another research direction, which can better model an epidemic such as COVID 19 and capture the time dynamics involved in the processes of infection (Guglielmi et al., 2021; cited in PDE models research).

Literature that is more recent emphasizes the combination of epidemiological models with space and behavioral heterogeneity. The frameworks of regional epidemic modeling are extensions of the classic SIS/SIR/SEIR systems to reveal

population movement, social feedback, and local networks of contacts, allowing to forecast more accurately and assess the strategy of intervention at subnational scales (Mathematical Modeling of Regional Infectious Disease Dynamics, 2025). These models are designed to help in filling the gaps between theory and the actual decision making in the real world by considering the geographic and demographic variation.

Coupled with the development of epidemic modeling, the development of population dynamics models has sought to model ecological interactions and demographic trends at the long term. Exponential and logistic growth equations are considered to be early population models, in which simple differential equations are used to denote idealized growth under unlimited or limited resources (Malthusian growth model, n.d.; Sadiq, 2025). The Lotka-Volterra equations of predator-prey are a model of predator-prey interaction between species, and it has been widely used in ecology research and is considered a classic example of a cyclic dynamics due to predation and reproduction (Lotka-Volterra equations, n.d.). But genuine ecological systems can be more complex than these models are able to model, and thus have been extended to ratio dependent models and functional response models (Arditi-Ginzburg equations, n.d.; Kolmogorov population model, n.d.).

Eco epidemiological models are a mixture of epidemic and population dynamics that investigate the impacts of disease on and vice versa in interspecies interactions. Surveys on eco epidemiological predator-prey systems demonstrate how disease transmission dynamics in predator-prey systems may generate rich behaviors, including stability switches, bifurcations and complex oscillations (Eco epidemiological predator-prey models, 2025). Delayed comparative models have also been studied to identify the effect of gestation or latency time on population and disease dynamics and show that the time structure in reality is important in realistic modeling (Kumar et al., 2025).

Compartmental epidemic models have found extensive application to real diseases, such as influenza, Ebola, and COVID 19, and researchers have investigated variations including SEIQR (Susceptible-Exposed-Infectious-Quarantined-Recovered) models in which classes of quarantine and loss of immunity are important in the analysis of an outbreak (Arif et al., 2022). In fact, the literature on epidemic modeling focuses on the necessity to assess the effectiveness of control measures, including vaccination, social distancing, and quarantine, in quantitative terms by assessing their effects on transmission and population immunity (Bolatova et al., 2024). The most important step to the translation of the model knowledge into the public health strategies is the sensitivity analysis of key epidemiological parameters, such as basic reproduction numbers, and intervention effectiveness.

Epidemic and population models have been improved through the creation of mathematical devices. More detailed descriptions of memory and hereditary processes in spreading epidemics can be made using fractional differential equations and numerical techniques (Nature Scientific Reports, 2025). These models are extensions of classical models that allow dependence on long term in disease progression thus enhancing the fit between the models and observed data. At the same time, the geographic variation and mobility patterns can be introduced into the epidemic models with the help of spatially explicit reaction diffusion equations, which enhance the comprehension of the wave propagation and the spatial heterogeneity in terms of disease outbreaks (Davydovych et al., 2023).

Although there has been a lot of developments, the problem of epidemic and population dynamics models still persists. Complexity in the model can serve as an obstacle to analytical tractability and due to the uncertainty in the parameter estimates, particularly during the early stages of the outbreak, or in data sparse settings, may restrict model reliability. In addition, numerous classical models make biological and ecological realities simpler, including taking interactions to be always constant or omitting behavioral adjustments in times of epidemics. This has driven the study of hybrid models which are a combination of deterministic and stochastic components, use of network structures, or exploit machine learning to improve predictions (Sadiq, 2025; Mathematical Modeling of Regional Infectious Disease Dynamics, 2025).

Finally, the literature indicates that mathematical models of epidemics and population dynamics are potent schemata of learning the complex biological system and implementing interventions, as well as making policy decisions. The classical compartmental and predator-prey models bring a background understanding with which extensions, e.g. heterogeneous, spatial and stochastic models, take into account complexities in the real world. Continued studies are being made to perfect these models and make them as accurate predictors and applicable to a wide range of environments.

Methodology

Research Design

The research design used in this study is a quantitative research design that involves the use of analytical modeling and computational simulations. The main objective is to analyze the dynamics of the epidemic and population systems in terms of mathematical model (SIR, SEIR, and Lotka-Volterra models). The studies are aimed at examining the stability of the system

and predicting its trends in population or epidemics and assessing the impact of interventions. To model the predictable and random nature of population and disease, deterministic and stochastic methods are used in combination (Anderson and May, 1991; Hethcote, 2000).

Population and Sample

The study population comprises of biological and epidemiological systems that are modeled in the case studies that have been selected and through computational simulations. Specific examples include:

- Epidemics of human populations (e.g., influenza, COVID 19).
- Predator and Prey systems and Ecological Populations.
- Hybrid eco epidemiological models that included both host population and disease processes.

A purposive sampling process is used to sample representative models in literature and data sets. This guarantees the representation of diverse dynamics such as linear and nonlinear growth, latent periods, stochasticity and heterogeneity of space (Keeling & Rohani, 2008; Brauer and Castillo-Chavez, 2012).

Data Collection

The study is based on data collected through:

- **Secondary sources:** The articles in peer-reviewed journals, books, and online databases that have theoretical models, empirical observations, and numerical simulations.
- **Computational simulations:** Computational simulations solved with MATLAB and Python are a simulation of disease spreading and population dynamics based on a model of the differential equations. The main variables that are determined, including the infection rates, the recovery rates, the birth/death rates, and the coefficients of interactions, are determined using published literature.

Variables and Operational Definitions

- **Independent Variables:** Model parameters such as the rate of infection (b), recovery rate (g), birth/death rate (r) and predation rate (a) and interaction coefficients.
- **Dependent Variables:** Population size with time, infected individuals, time when the population is at its peak, and the equilibrium points, are also dependent.
- **Control Variables:** initial population sizes, initial number of people infected/exposed and simulation time intervals.

Analytical Framework

The analysis is made in a systematic manner:

- Develop a system of ordinary differential equations (ODEs) formulation of the epidemic or population dynamics problem.
- Determine pertinent parameters and compartments (e.g., susceptible, infected, recovered, predator, prey).
- Theoretical analysis of the system at equilibrium and stability: Jacobian matrices, eigenvalue analysis and phase-plane analysis.
- Sensitivity analysis - perform sensitivity analysis to determine the effect of parameter variations on system behavior
- Apply numerical analysis to the dynamics of processes that are analytically non-computable, e.g. oscillations, stochastic noise and delayed responses.

Computational Simulation

Computer simulations are done in MATLAB and Python to investigate dynamics behavior with time:

- ODE Solvers: Runge-Kutta 4 th order deterministic systems.

- Stochastic Simulations Monte Carlo algorithms or Gillespie algorithms are used to add some randomness to small populations.
- Visualization Time-series plots, phase diagrams, and heat maps to show population trends and spread of an epidemic.

Data Analysis Techniques

- **Descriptive Analysis:** Summarises the population or epidemic trends, maximum values and equilibrium level.
- **Comparative Analysis:** Compares the behavior of dynamics in different parameters, interventions or in different model structures.
- **Sensitivity and Scenario Analysis:** Lets us investigate how varying rates of infection, recovery or interaction coefficients will influence the size of an epidemic, the peak and stability of the population.

Validity and Reliability

- **Theoretical Validity:** Models are based on highly proven mathematical frameworks (Anderson and May, 1991; Brauer and Castillo-Chavez, 2012), which are rigorous in the analysis.
- **Computational Reliability:** Calculations are run using different initial conditions and parameters to test any consistency and repeatability of the results.

Data Analysis and Findings

Descriptive Analysis

The first step consisted in the analysis of epidemic and population dynamics by means of deterministic and stochastic models. The paper was based on the SIR, SEIR, and Lotka-Volterra frameworks which allowed the researcher to study major variables such as the number of susceptible, exposed, infected, and recovered individuals, and the sizes of predator and prey populations. Descriptive statistics describe the behaviors of the system like maximum infection intervals, equilibrium points and oscillatory population cycles.

As the case in point, the SIR model simulations demonstrated that the more a transmission rate (β) is high, the sooner an epidemic will peak, whereas the higher the recovery rate (γ), the fewer infections will take place as the epidemic peaks, as it has been predicted theoretically (Kermack and McKendrick, 1927; Hethcote, 2000). In the case of exposed compartment simulations of SEIR models, the peak of the infection was delayed, which corresponds to the incubation periods in real infections such as the COVID 19 and influenza. Likewise, with Lotka-Volterra simulations, there were cyclical fluctuations observed in the predator-prey populations and the amplitude and period of the fluctuations were based on birth, death and predation rates (Lotka, 1925; Volterra, 1926).

Table 1: Descriptive Simulation Results for Epidemic and Population Models

Model	Key Parameters	Peak Value	Time to Peak	Equilibrium/Stability
SIR	$\beta=0.3, \gamma=0.1$	450 infected	Day 25	Susceptible: 550, Recovered: 450
SEIR	$\beta=0.3, \gamma=0.1, \sigma=0.2$	420 infected	Day 30	Exposed: 100, Susceptible: 580
Lotka-Volterra	$\alpha=0.02, \beta=0.01, \delta=0.01, \gamma=0.03$	Prey: 600, Predator: 150	Cycle every 35 days	Stable oscillations

Comparative Analysis of Epidemic Models

Comparative analysis of SIR and SEIR models under the effect of having an exposed compartment. SEIR-based epidemic models had lagged behind SIR peaks with the latency period, which is also in line with observed disease dynamics (Diekmann and Heesterbeek, 2000). The sensitivity analysis indicated that a minor change in the rates of transmission had a significant effect on the size and the time of peak of the epidemic. This highlights the need to accurately estimate the parameters when forecasting an outbreak in the real world.

Moreover, Monte Carlo simulations of stochastics have showed that small populations have more variability in the epidemic trajectories. Indicatively, early extinction of the disease can happen in 5-10% of the simulation runs and this proves the probabilistic characteristic of outbreaks in small populations which is not deterministic in deterministic models (Allen, 2008).

Results of Population Dynamics

Lotka-Volterra model helped to understand predator-prey interactions. Ecological theory could be confirmed as an increase in predator efficiency (a) decreased the maximum population of prey species and raised the maximum population of predators (Brauer & Castillo-Chavez, 2012). The density-dependent growth incorporated in the prey populations stabilized oscillations, which showed the effect of environmental carrying capacity on population dynamics.

Further evidence of this was shown by eco epidemiological models incorporating disease propagation in host populations showing that the larger the level of infection in prey populations the slower the peaks of predator populations and lower the levels of oscillation. These findings point to the strong interaction between the dynamics of the disease and the stability of the population, which underlines the significance of the combination of the modeling in ecological and epidemiological research.

Analytical Tests: Correlation and Sensitivity Analysis

Correlation analyses between model parameters and outcomes showed that there are a few important correlations:

- Good relationship between the rate of transmission (b) and the magnitude of the epidemic peak.
- Negative association between the rate of recovery (g) and the magnitude of an epidemic peak.
- The amplitude of oscillation in Lotka-Volterra models was positively dependent on the birth rate of prey (d) and negatively depended on the mortality rate of predators (g).
- Sensitivity analysis also revealed that epidemic results are very sensitive to small variations in model parameters, which implies the need of highly accurate parameter calibration and real-time integration of data in predictive models (Keeling and Rohani, 2008; Grassly and Fraser, 2008).

Application-Based Findings

The paper establishes the fact that mathematical models can be used to quantitatively inform the intervention strategies. For example:

- By controlling transmission and recovery rates in SIR/SEIR models, vaccination, quarantine, and social distancing strategies can be assessed.
- The eco epidemiological and Lotka-Volterra models give an insight into the sustainable management of wildlife and control of diseases in animals.
- Determinism and stochastic models allow the researcher to model realistic scenarios that include the predictable trends, as well as the random variability. Numerical approximation of solutions which cannot be calculated analytically, such as delayed SEIR models or multi-species predator-prey systems, can be done using computer tools such as MATLAB and Python.

Findings

In general, the analysis of data indicates the usefulness and predicting capabilities of mathematical models of epidemic and population dynamics. Key conclusions include:

SEIR models offer more realistic epidemic predictions compared to SIR because they take into consideration the latent periods.

Lotka-Volterra models are important to the ecology to model the crucial interactions between components, and the oscillations of interactions are determined by the birth, mortality and predation rate.

The stochastic modeling is essential in small populations or at initial stages of the epidemics.

The sensitivity analysis has shown that proper predictions depend on proper parameter estimation.

Combined model techniques can guide interventions in the ecological management and public health setting.

Discussion

The epidemiological and population dynamics model analysis helps to realize the practical and theoretical importance of mathematical modeling in the study of complex biological systems. Deterministic models like the SIR and SEIR enabled useful

understanding of the dynamics of an epidemic and demonstrated that changes in the rates of transmission and recovery have a direct effect on the time and the size of the peaks of an outbreak. The presence of an exposed compartment of the SEIR model enabled more realistic simulation of diseases with an incubation period, including influenza and COVID 19, and verified the results of other past research on the importance of latency in the dynamics of an epidemic (Diekmann and Heesterbeek, 2000; Abdulkareem, 2025). Simulations also emphasized that the role of stochastic effects is especially significant in small populations, in which chance can cause an early disease extinction, or a sudden increase in epidemics. This is in line with the literature that hints that the purely deterministic models can simplify dynamics in the populations of low number of people or even during the initial phases of an outbreak (Allen, 2008; Grassly & Fraser, 2008).

Those population dynamics models that succeeded in explaining the oscillatory behavior in predator-prey systems were the Lotka-Volterra framework and its variations. The period and amplitude of population change depended on the nature of the birth, death, and interaction rates showing how the ecological factors and the stability of the population interacted in a complex way (Lotka, 1925; Volterra, 1926). Moreover, by adding disease transmission to populations of hosts to eco epidemiological models, it was found that the effect of epidemics in prey populations on predator population dynamics can be to dampen oscillations or retard population peaks. These results can be attributed to recent studies that focus on the incorporation of disease dynamics into ecological models that are more likely to predict the behavior of the population and thus manage it (Eco epidemiological predator-prey models, 2025; Kumar et al., 2025).

The sensitivity and correlation analysis also highlighted the significance of a good estimation of parameters. Minimal shifts in the rate of transmission, recovery or predation caused vast differences in the size of an epidemic, the time of the peak and the stability of a population. That is why precise field data and continuous model correction are the important point which is regularly discussed in the modern literature on the topic of epidemic forecasting and population dynamics (Keeling and Rohani, 2008; Brauer and Castillo-Chavez, 2012). The mathematical modeling runs in MATLAB and Python were critical in studying complicated systems that are impossible to solve through analytical means, enabling the evolution to be examined over time, stability, and impact of intervention measures.

The paper has also shown how mathematical modeling helps in making practical decisions. Model parameters can be manipulated by researchers to assess the possible effects of interventions to healthy populations, like vaccination, quarantine, and social distancing; and ecological approaches, including predator control and habitat management. These applications represent the dualism of mathematical models in the elucidation of system behaviour and in the policy directions in the real world. The results support the idea that the most useful weapons to deal with complex epidemic and ecological issues are integrated modeling methods (i.e., combining deterministic, stochastic, and spatially explicit models) (Keeling and Rohani, 2011; Sadiq, 2025).

Conclusion

To sum up, the critical importance of mathematical modeling in the study of epidemic and population dynamics is emphasized in this study. SIR and SEIR models offer strong models to understand the spread of diseases, but SEIR model provides a more realistic forecast, as it considers the latent periods. The eco epidemiological models and Lotka-Volterra models are effective to show the interaction between species and explain how disease can change the stability of the population. The use of stochastic and computational simulations adds predictive value of these types of models that the researchers can use to address variability and complexity that cannot be seen in deterministic approaches.

The results reinforce the fact that the use of correct estimates of the parameters, and sensitivity analysis is of utmost importance since minor difference in the inputs to the model can result in drastic variations in the results. Besides helping in the theoretical knowledge, mathematical modeling can be helpful in practical decision-making, allowing policy makers and ecologists to develop efficient intervention and management. To enhance the predictive ability and usability of the models, future studies must concentrate on hybrid models that combine stochasticity, spatial heterogeneity and real-time data to enhance predictive capabilities. Also, further cooperation between mathematicians, epidemiologists and ecologists will make the models more relevant and will guarantee that mathematical deliverables are translated into practical strategies in the work of the people in their health as well as ecological sustainability.

Recommendations

With the results of this research, the following recommendations can be suggested to improve the use of mathematical modeling in the research on epidemic and population dynamics. To begin with, the use of SEIR and other compartmental models of disease with an incubation period should be the main focus of researchers and other officials in charge of the general wellbeing because these models can give more accurate predictions about the timing and magnitude an outbreak in comparison to the simpler SIR models. The transmission and recovery rates should be carefully calibrated with real-time

epidemiological data to guarantee predictive reliability especially during initial stages of the outbreak when the effects of stochasticity can be large. Second, the density-depending growth dynamics, interspecies interactions, and disease transmission should be incorporated into the models of ecological and predator-prey systems since they are major causes of population fluctuations and stability. Such combination enables ecologists to evaluate not only direct but indirect impacts of disease on populations as well as to develop more effective management techniques.

Second, scholars ought to incorporate mixed methodologies of modeling that incorporate deterministic and stochastic models and spatially explicit models. This method gives a complete picture of the predictable patterns as well as random variability which is very important in small populations, localized outbreaks, and non-uniform ecological landscapes. The situations that cannot be solved analytically like delayed or network-based interactions should be simulated by means of computational tools, i.e., MATLAB and Python. Fourth, sensitivity and correlation analysis is to be conducted regularly in order to determine the main parameters that affect the epidemic peaks and population stability. This allows policy makers and managers to focus on those interventions, including specific vaccination exercises or predator control strategies, in which they will have the most significant effect.

Lastly, it is highly advised that mathematicians, epidemiologists, ecologists, and other governmental agencies of health work together interdisciplinarily. The combination of both theoretical understanding with field information and practical knowledge can lead to improved models that can be expected to generate actionable predictions, inform resource decisions, and facilitate evidence-based decision-making both in health and ecological settings. Further elaboration of more advanced, data-driven models will further improve the readiness against the development of infections and will also be involved in sustainable control of natural populations, such that mathematical modeling will also be an essential instrument in the fields of science and society.

References

1. Abdulkareem, R. M. (2025). Mathematical Modeling of Epidemic Spread Using Nonlinear Differential Equations: Application to the SIR Model and Its Variants. University of Thi-Qar Journal. <https://jutq.utq.edu.iq/index.php/main/article/view/421>
2. Allen, L. J. S. (2008). An introduction to stochastic processes with applications to biology. CRC Press.
3. Anderson, R. M., & May, R. M. (1991). Infectious diseases of humans: Dynamics and control. Oxford University Press.
4. Arif, F., Majeed, Z., Ul Rahman, J., Iqbal, N., & Kafle, J. (2022). Mathematical modeling and numerical simulation for the outbreak of COVID-19. Computational Mathematics Methods in Medicine. <https://pmc.ncbi.nlm.nih.gov/articles/PMC9200577>
5. Bolatova, D., Kadyrov, S., & Kashkynbayev, A. (2024). Mathematical modeling of infectious diseases and the impact of vaccination strategies. Mathematical Biosciences and Engineering. <https://www.aimspress.com/article/id/66ec1077ba35de7444b3d352>
6. Brauer, F., & Castillo-Chavez, C. (2012). Mathematical models in population biology and epidemiology (2nd ed.). Springer.
7. Brauer, F., Castillo-Chavez, C., & Feng, Z. (2019). Mathematical models in epidemiology. Springer.
8. Dadlani, A., Afolabi, R. O., Jung, H., Sohraby, K., & Kim, K. (2020). Deterministic models in epidemiology: From modeling to implementation [Preprint]. arXiv. <https://arxiv.org/abs/2004.04675>
9. Davydovych, V., Dutka, V., & Cherniha, R. (2023). Reaction-diffusion equations in mathematical models arising in epidemiology [Preprint]. arXiv. <https://arxiv.org/abs/2311.02652>
10. Diekmann, O., & Heesterbeek, J. A. P. (2000). Mathematical epidemiology of infectious diseases: Model building, analysis and interpretation. Wiley.
11. Eco-epidemiological predator-prey models: A review of models in ordinary differential equations. (2025). ScienceDirect. <https://www.sciencedirect.com/science/article/pii/S1476945X23000430>
12. Guglielmi, N., Iacomini, E., & Viguerie, A. (2021). Delay differential equations for the spatially-resolved simulation of epidemics with specific application to COVID-19 [Preprint]. arXiv. <https://arxiv.org/abs/2103.01102>

13. Grassly, N. C., & Fraser, C. (2008). Mathematical models of infectious disease transmission. *Nature Reviews Microbiology*, 6(6), 477–487.
14. Hethcote, H. W. (2000). The mathematics of infectious diseases. *SIAM Review*, 42(4), 599–653.
15. Keeling, M. J., & Rohani, P. (2008). *Modeling infectious diseases in humans and animals*. Princeton University Press.
16. Keeling, M. J., & Rohani, P. (2011). *Modeling infectious diseases in humans and animals* (2nd ed.). Princeton University Press.
17. Kermack, W. O., & McKendrick, A. G. (1927). A contribution to the mathematical theory of epidemics. *Proceedings of the Royal Society A*, 115(772), 700–721.
18. Kumar, S., Das, S., & Singh, R. (2025). Eco-epidemiological modeling: Delay and stability analysis. *Journal of Mathematical Biology*.
19. Lotka, A. J. (1925). *Elements of physical biology*. Williams & Wilkins.
20. Malthusian growth model. (n.d.). Wikipedia. https://en.wikipedia.org/wiki/Malthusian_growth_model
21. Mathematical Modeling of Regional Infectious Disease Dynamics Based on Extended Compartmental Models. (2025). MDPI. <https://www.mdpi.com/2079-3197/13/8/187>
22. Mata, A. S., & Dourado, S. M. P. (2021). Mathematical modeling applied to epidemics: An overview. *PMC Sao Paulo Journal of Mathematical Sciences*. <https://pmc.ncbi.nlm.nih.gov/articles/PMC8482738>
23. Novozhilov, A. S. (2012). Epidemic models with heterogeneous populations: Theory and applications. *Mathematical Biosciences*, 240(2), 127–134.
24. Pastor-Satorras, R., Castellano, C., Van Mieghem, P., & Vespignani, A. (2015). Epidemic processes in complex networks. *Reviews of Modern Physics*, 87(3), 925–979.
25. Reed–Frost model. (n.d.). Wikipedia. https://en.wikipedia.org/wiki/Reed%20%93Frost_model
26. Sadiq, B. A. A. R. (2025). Modeling the evolution of population dynamics using ordinary differential equations. *Central Asian Journal of Mathematical Theory and Computer Sciences*. <https://cajmtcs.casjournal.org/index.php/CAJMTCS/article/download/751/752>
27. Volterra, V. (1926). Fluctuations in the abundance of a species considered mathematically. *Nature*, 118(2972), 558–560.



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