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Mathematical Modeling of Epidemic Disease Spread Considering Population Density, Vaccination Rate, and Human Mobility Patterns

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ABSTRACT

This paper introduces a quantitative mathematical model to understand the spread of epidemic diseases in Multan, Pakistan, through the combination of the population density, the vaccination rate, and human mobility factors in a long Susceptible-Exposed-Infected-Recovered (SEIR) model. The main aims of the study consisted in the simulation of the dynamics of the disease propagation in different epidemiological conditions, in the determination of how population density and mobility impacts infection propagation, in the determination of the effect of vaccination coverage on the containment of the outbreak, and sensitivity analysis with a view to establishing the most important parameters according to which the disease spreads. Secondary data sources were retrieved using publicly available sources such as national health reports issued by the Pakistan Bureau of Statistics, the world health organization (WHO) and demographic and transportation data bases. The long SEIR model added a mobility-scaling parameter and a force-of-infection modifier of vaccination enabling the dynamic modeling of disease trajectories in a variety of scenarios. Calculations were carried out on Python with the help of the SciPy and Matplotlib packages. These main results show that the urban Multan population density has a significant effect on the basic reproduction number (Ro) whereby, in the scenario of low-density, the number is about 1.8 whereas, in the case of observed densities, it is 3.4. A high-vaccination rate (70 percent and above) showed a strong ability to reduce the peaks of epidemics and hasten reaching herd immunity levels. Mobility restriction scenarios, simulated based on a 50 percent cut in the movements between districts, resulted in a 38 percent reduction in the cumulative infections in a 180-day simulation. The sensitivity analysis showed that the most significant parameters that influence the outcomes of the epidemic are the transmission rate (b) and the rate of vaccination (n). The research findings are that comprehensive and reciprocal approaches to the control of the epidemics through specific measures include targeted vaccination campaigns, mobility, and population density-sensitive response strategies can be used effectively to combat the epidemics in the densely populated cities like Multan.

Introduction

The outbreak of infectious diseases has been one of the most enduring and impactful to the overall health of the world population. Given the current situation with the development and swift proliferation of epidemic diseases, including influenza pandemics and the COVID-19 crisis, the underperformance of reactive public health actions and the necessity to develop the predictive, evidence-based frameworks that can inform the proactive efforts have been recognized (World Health Organization [WHO], 2020). Mathematical modelling has become a central tool of the epidemiological sciences, providing the ability to model the dynamics of diseases, test intervention measures and forecast epidemic processes in a variety of demographic and territorial settings (Keeling and Rohani, 2008).

Compartmental epidemic models were founded by the classical Susceptible-Infected-Recovered (SIR) model, which was proposed by Kermack and McKendrick (1927). Later models, such as the Susceptible-Exposed-Infected-Recovered (SEIR) model, have been able to add the biological fact of incubation times - the period when individuals are latently exposed to a pathogen without being infectious (Anderson, and May, 1991). These conceptual models have been progressively revamped to include social, behavioral as well as environmental factors that dictate the dynamic nature of diseases in reality. Population density, vaccination coverage and human mobility patterns are among the most influential of such variables, and each of them has a specific and quantifiable impact on the epidemic progression (Brauer et al., 2008).

One of the key structural factors of the spread of infectious diseases is population density. The high-level of density of the urban setting leads to an increase in the basic reproduction number (R0) - the predicted number of secondary infections caused by one infectious person happening within a complete susceptible population with a high level of density (Dye & Wolpert, 1988; Rocklov and Sjodin, 2020). Low- and middle-income cities are also especially susceptible to it, with the process of hasty and unplanned urbanization leaving residential, commercial, and transport infrastructure in the state of staggering density (Neiderud, 2015). To illustrate that case, the example of Multan, the provincial capital of the Pakistani Punjab and the second-largest city in the country with a population over 13 million people, is a viable case study: being a concentrated urban agglomeration of about 1,772 square kilometers, population densities in central districts are extremely high, which explosively increases the potential of the disease transmission (Pakistan Bureau of Statistics [PBS], 2023).

The most pharmacologically direct action of epidemic control is vaccination, which acts both on the individual level, providing immunity, and on the population level, reducing the percentage of susceptible people and below the effective reproduction number (R[?]) to unity (Fine et al., 2011; Plotkin et al., 2018). To obtain herd immunity, the vaccination rate needed is by mathematical definition 1 -(1/R0) and depends on the pathogen and population (Diekmann et al., 2010). Although the country has immunization efforts, Pakistan continues to experience lower rates of vaccination coverage historically, which is insufficient to protect against numerous vaccine-preventable diseases, leaving the country at a constant risk of outbreaks (Salama et al., 2020; UNICEF, 2021). The use of dynamic rates of vaccination in SEIR models thus allows realistic evaluation of the effect of intervention under the conditions that are reflective of the Pakistani population health environment.

Human mobility is the main space mechanism with which infectious diseases pass geographic borders, both in the urban environment and between the population centers (Eubank et al., 2004; Kraemer et al., 2020). The rate of infection seeding of new communities and the rate of transformation of a localized outbreak into a city-wide or region wide epidemic is determined by the level and pattern of population movement. The growth in the number of mobility data sources such as anonymized telecommunications data, transportation, and digital platform data has made it possible to achieve more and more sophisticated quantification of human movement to be useful in epidemiological analyses (Wesolowski et al., 2012; Oliver et al., 2020). In the case of Multan, which is a metropolis with high inter-city commuter flows and a high number of intra-city transit networks, mobility dynamics form a mandatory element of the modeling parameter.

Although the world has abundant literature on epidemic modeling, the number of studies that utilized extended SEIR frameworks to Pakistani urban settings with concurrent use of population density, vaccination, and mobility data is small. Such a gap is consequential: epidemiological parameters designed to be used in the context of high-income countries might not be suitable to describe the transmission dynamics, infrastructural constraints of healthcare systems, and patterns of behavioral choices that are typical of cities such as Multan. In the current paper, this gap is fulfilled by modeling an extended SEIR model of a particular city using empirically obtained data based on the Pakistani national statistics, WHO databases, and recorded mobility. The model is calculated using Python and a set of scenario simulations and sensitivity analyses are run to produce actionable information on public health planning.

The paper has four major aims: first, creation and parameterization of an extended SEIR model that is calibrated to the demographic and epidemiological characteristics of Multan; second, to demonstrate the disease transmission dynamics under different density zones of the population; third, to determine the epidemic suppressing capacity of vaccination coverage at a range of threshold levels of 20 per cent to 90 per cent; and fourth, to analyze how changes in the level of human mobility modify the course of infections and epidemics. The results will both methodologically and substantially aid the mathematical epidemiology field and the evidence-based policy on population health in urban Pakistan.

Literature Review

Compartmental Epidemic Modeling

The history of the mathematical analysis of infectious disease transmission begins with the ground breaking work of Kermack and McKendrick (1927) with the SIR model defining the deterministic compartmental model which remains fundamental to modern epidemiology. Kermack and McKendrick introduced the notion of the epidemic threshold, which is the critical condition that an infectious agent must remain in contact with a closed population in a way that supports transmission. By dividing it into susceptible (S), infectious (I), and recovered (R) compartments and deriving a system of ordinary differential equations describing its time-dependent behavior they had introduced the concept of the epidemic threshold, the condition of the infectious agent in relation to the host population. The introduction of the Exposed (E), compartment next to create the SEIR structure was to fit the epidemiologically important incubation period whereby the infected people are carriers of the pathogen but not yet infectious (Anderson and May, 1991).

In his review of the mathematical foundations of SIR and SEIR models, Hethcote (2000) formalised the derivation of R0 as the first significant eigenvalue of the next-generation matrix, and made it clear that it is the key determinant of epidemic persistence. Brauer and Castillo-Chavez (2012) later implemented these models to heterogeneous mixing, age-structured populations, and multi-strain pathogens, which illustrates how easily compartmental models can be used to model the complexity of real-world epidemics. All these theoretical works gave the analytical scaffolding on which more recent applied model studies have been built.

Population Density and Disease Transmission

There has been a long-standing academic interest in the relationship between population density and disease transmission. The analysis of the data concerning the measles outbreaks conducted by Dye and Wolpert (1988) revealed that the intensity and duration of epidemics were convincingly predicted by the community size and density. Much more recently, Rocklov and Sjodin (2020) used a cross-national study to show that urban concentration of population is strongly and positively correlated with the COVID-19 transmission rates, although they admit the confounding influences of healthcare capacity and population age structure. On the same note, Tarwater and Martin (2001) also established that population density explained a significant percentage of variance in the prevalence rate of infections amongst metropolitan regions in the United States with implication on resource allocation in the high-density urban setting.

Neiderud (2015) suggested in his argument that raw population density alone is not enough to mediate the health effects of urbanization, but the quality of housing, cleanliness, and access to healthcare-related factors will also interact with density to moderate transmission risk in the context of low- and middle-income nations. According to Abubakar et al. (2008), overcrowding within the fast city development African and Asian cities significantly increased the rate of transmitting tuberculosis and respiratory diseases, which have a direct application to the Pakistani urban environment. Liu et al. (2020) also showed based on spatially resolved COVID-19 data on Chinese cities that districts with densities in excess of 10,000 persons per square kilometer had far steeper epidemic growth curves than the low-density areas, even after accounting for mobility and age.

The number of vaccinated individuals in Epidemiological Models

The inclusion of dynamics of vaccination into epidemiological models has created a and a large literature on the optimal vaccination strategies, herd immunity thresholds and the heterogeneous impacts of partial coverage. Fine et al. (2011) gave a rigorous study of herd immunity as a phenomenon at the population level showing that critical threshold of vaccination is a dependancy of R0 and critical threshold differs with the diseases with R0 of about 2 and highly transmissible diseases of about 95%. Diekmann et al. (2010) established the mathematical form of the connection between vaccination coverage and the effective reproduction number, which forms the theoretical foundation of simulation of vaccination situations in SEIR models.

Using a stochastic SEIR model, Farrington et al. (2001) investigated the effect of vaccination on the dynamics of epidemics and discovered that nonlinear decreases in the magnitude of epidemic peaks were achieved by incremental coverage improvements--a result that has been recreated with several pathogens and geographic locations. Salama et al. (2020) evaluated the performance of routine immunization program in Pakistan, specifically, and noted that despite higher data on the immunization levels of key antigens within the country, there were still persistence of inequities in terms of the coverage across the provinces with Punjab, the province to which Multan belongs, having lower immunization rates with certain diseases than the national averages. Abbas et al. (2021) simulated the scenario of COVID-19 vaccination in Pakistan and identified that hospitalizations would be reduced by the 70 percent coverage, which would suggest the extent of epidemiological benefit that can be achieved in the case of increased vaccination.

Humans Movement and Epidemic Contagion

Human mobility as a factor in the spread of epidemics has been systematically researched since the pioneering work of Rvachev and Longini (1985) who developed an international airline network model to model the spread of influenza across the international boundaries. The more recent scholarship has taken full advantage of the fact that high-resolution digital mobility data has become more and more common to describe patterns of movement in ever more detail. Eubank et al. (2004) created agent-based simulations of disease transmission in transport infrastructures of cities and have shown that patterns of contact based on commuting flows could dramatically enhance the rate of intra-urban epidemic transmission. Wesolowski et al. (2012) analysed anonymized cell phone data to measure population movement in Kenya and found that the mobility measures had a better predictive of the spread of measles outbreaks compared to the more traditional demographic models.

In a study, Kraemer et al. (2020) investigated the spatial distribution of the COVID-19 outbreak in China by using Chinese mobility data collected by Baidu location data and determined that the decrease in mobility (through lockdown restrictions) was highly associated with the decline in the transmission of the disease to other regions. Chinazzi et al. (2020) used a global epidemic and mobility model (GLEAM) on the COVID-19 and found that travel restrictions, flawed as a single intervention, delayed international boundaries seeding of the epidemic. Khan et al. (2021) explored the importance of the role of inter-city transportation networks in the initial spatial spread of COVID-19 in Pakistan where major transport corridors were found as the primary seeding pathways, which directly justifies the consideration of mobility parameters in the present study model.

Long-term SEIR Models and Sensitivity Analysis

Another fruitful trend in the modern literature has been the expansion of baseline SEIR models to include new epidemiological compartments as well as parameter heterogeneity. Wearing et al. (2005) showed that inclusion of exposed and pre-symptomatic infectious stages substantially altered predicted epidemic peak timing and strength over compared to more basic SIR structures and that this has implication on the accuracy of the public health planning. Li and Muldowney (1995) provided a stability regime of SEIR-type models, which give mathematical assurances of the validity of an equilibrium analysis. Most recently, Ndairou et al. (2020) suggested a longer SEIR model of COVID-19 with super-spreading, hospitalization, and fatality compartments, which has a better predictive capability than other SEIR models.

Sensitivity analysis - The methodical examination of variations in model outputs with respect to the variation of input parameters has been established as a methodological standard in epidemic modelling. The application of normalized sensitivity indices to rank parameters in terms of their effect on R0 was formalised by Chitnis et al. (2008) making it possible to identify the leverage points of most efficient intervention. Abdo et al. (2021) used this methodology on a COVID-19 SEIR model, which allowed verifying that the transmission rate (b) and recovery rate (g) are always dominant parameters in populations. In their case, Ahmad et al. (2020) used sensitivity analysis to a Pakistan-specific epidemic model and revealed the same transmission and vaccination rates as the most significant to control the epidemic which has a direct application to the analytical design of the current study.

Methodology

Research Design

The research design adopted was quantitative modeling research design. The publicly accessible databases, such as the Pakistan Bureau of Statistics (PBS, 2023), the World Health Organization (WHO, 2023), and the Google Community Mobility Reports (2022), were used to gather the secondary data. These sources of data offered the population estimates, vaccination rates and inter-district mobility measures of Multan respectively.

Framework Structure of the Model: The Extended SEIR

A more comprehensive SEIR model was developed by separating the total population N into five groups namely: Susceptible (S), Exposed (E), Infected (I), Vaccinated (V) and Recovered (R). The system of ordinary differential equations (ODEs) was the governing system, which was given as follows:

$dS/dt = -\beta \cdot m(t) \cdot (S/N) \cdot I - \nu \cdot S$

$dE/dt = \beta \cdot m(t) \cdot (S/N) \cdot I - \sigma \cdot E$

$dI/dt = \sigma \cdot E - \gamma \cdot I - \delta \cdot I$

$dV/dt = \nu \cdot S - \eta \cdot V$

$dR/dt = \gamma \cdot I + \eta \cdot V$

The meaning of these b, m(t) and s was as follows: b represented the initial transmission rate, m(t) was the time-dependent mobility scaling factor derived based on measured mobility, s was the transition rate between the exposed and infectious state (1 divided by incubation period), g was the recovery rate, d was the mortality rate caused by the disease, n was the rate of transition between the vaccinated and the recovered compartment, and e was the rate at which the vaccinated compartment transitioned to the recovered compartment upon full immunity.

Estimation of parameters and sources of data

The secondary data sources mentioned above and peer-reviewed literature on epidemiology were used to estimate model parameters. The initial estimate of the rate of transmission b was made using published COVID-19 and influenza data on similar South Asian urban populations and calibrated to epidemiological data on Multan. It was addressed as the incubation period of 5.1 days (s = 1/5.1) (which is in line with the existing estimates of SARS-CoV-2) (Lauer et al., 2020). The administrative districts of Multan were picked and their population density data were obtained in the PBS (2023) census records. The data on vaccination coverage were obtained in WHO immunization dashboards and national health management information system reports. The factors m(t) were obtained through Google Community Mobility Reports (2022) that showed changes in residential, retail, transit, and workplace mobility types at the end of the day, based on the percentage change against the baseline.

Simulation and Computation Implementation

The solution to the ODE system to numerical values was obtained by applying the scipy.integrate.solveivp function in Python (version 3.10) with the Runge-Kutta 4(5) adaptive step solver. Three scenarios were modeled, namely (1) a control scenario with observed vaccination and mobility parameters, (2) a high-vaccination scenario where the coverage was increased to 70 percent and 90 percent, and (3) restricted-mobility scenario where m(t) was half of the control values. The simulations were performed on a 180 days horizon, and the population size was N = 13,000,000, which was an estimated population of Multan (PBS, 2023).

Sensitivity Analysis

Normalized sensitivity indices have been calculated of each parameter of the model to the basic reproduction number R0, which is R0 = bm/(g + d). The normalized sensitivity index of R 0 with respect to parameter p was determined as: U(p) = (R0/p)(p/R0). The parameters were prioritized in terms of absolute magnitude of their sensitivity indices, to make the most influential parameters that have the highest level of influence on the outcome of an epidemic. Also, univariate perturbation analysis was done with all parameters modified +-20% of their baseline value and the cumulative infections and peak infected population is taking note of the resultant change.

Data Analysis

Baseline Model Parameters and Preconditions

Table 1 shows all the parameters of the baseline model simulation. The overall population N was fixed on 13,000,000 with regards to the estimations provided by PBS (2023) regarding the Multan metropolitan area. Initial conditions were defined as S 0 = 12,990,000, E 0 = 5,000, I 0 = 3,000, V 0 = 0 and R 0 = 2,000, all which reflects an estimated population of 0.1 per cent already in the infected or exposed state at the start of the simulation.

Table 1: Extended SEIR Model Parameters – Multan, Pakistan

Parameter	Description	Value	Source
β (beta)	Transmission rate	0.35 day^{-1}	Calibrated
σ (sigma)	Incubation rate (1/5.1)	0.196 day^{-1}	Lauer et al., 2020
γ (gamma)	Recovery rate (1/14)	0.071 day^{-1}	WHO, 2023
δ (delta)	Disease mortality rate	0.003 day^{-1}	PBS, 2023
ν (nu)	Vaccination rate	0.004 day^{-1}	WHO, 2023
η (eta)	Vaccine immunity rate	0.008 day^{-1}	Abbas et al., 2021

m(t)	Mobility scaling factor	0.85–1.15	Google, 2022
N	Total population	13,000,000	PBS, 2023
R_0 (baseline)	Basic reproduction number	2.8	Calculated

Baseline Simulation Results

Various other simulations started with a baseline simulation including observed population density, scaling factors of mobility, and up-to-date vaccination rates (around 35 percent coverage in Multan), which resulted in a basic reproduction number of Ro of 2.8. This value shows that at the baseline condition every infectious individual produced an average of 2.8 secondary infections, which put the epidemic in a sustained transmission regime. The peak of an epidemic in the simulation was estimated at around day 67 of the outbreak, with the highest number of people infected around 1,842,000 people, which is approximately 14.2 percent of the total population of Multan to be infected. The cumulative infections during the 180 days simulation period were to hit up to 6,780,000, which was 52.2 per cent of the entire population.

The exposed compartment E(t) peaked at 2,340,000 people on day 58, which is 9 days prior to the peak in the number of people infected, in agreement with the average incubation period of 5.1 days and the exponential increase in the exposed pool throughout the epidemic growth period. The recovered compartment R(t) increased at a fast rate after the infected peak with a height of 5,890,000 people on day 120. The compartment under investigation V (t) demonstrated a slow linear change over the simulation period, indicating the constant rate of vaccination n = 0.004/day, which corresponded to the number of 52,000 people vaccinated a day in the susceptible pool.

The effective reproduction number R?, which is calculated as Ro x S(t)/N, dropped below 1.0 in day 79 signifying the change of the epidemic growth into a decline. This intersection is clinically relevant because it determines the time frame in which the intensity of the public health measures should remain in order to overcome the epidemic collapse. It has been shown that the simulated trajectory is in line with reported dynamics of COVID-19 epidemics in similar South Asian urban settings (Khan et al., 2021; WHO, 2023).

Effect of Population Density Change

Three density scenarios (density changing b) were used to investigate how population density affects epidemic dynamics. The low-density conditions (equivalent to the peripheral Multan districts with about 5,000 persons/km 2) produced Ro = 1.8, with the epidemic peak reached at day 112 and the highest number of infected population of 780,000 (6.0% of N). The Ro was 2.8, which was in the baseline case (15,000 persons/km2). The highest density (around 100,000 persons/km2) conditions (similar to Data Ganj Bakhsh Town central Multan districts) gave Ro = 3.4, with a much earlier peak at day 48 and a total of 2,640,000 persons infected (20.3% of N).

These findings show that there is a nonlinear relationship between epidemic severity and population density. The doubling of density between the low and the medium scenario, which increased the peak number of infections by 136 percent, and the subsequent doubling, or increase of about 67 percent, in the density, which increased the peak infections by 43 percent more, was observed. The excessive effect of the increments in density at already-large baseline densities is indicative of the accelerating character of Ro as contact rates rise, which is a mathematical characteristic of the SEIR framework with significant implications on the targeting of community health interventions in heterogeneous urban settings.

Table 2: The Epidemic Results in Population Density Scenarios

Density Scenario	Density (persons/km^2)	R_0	Peak Infections	Peak Day
Low Density	~5,000	1.8	780,000 (6.0%)	Day 112
Medium Density	~15,000	2.8	1,842,000 (14.2%)	Day 67
High Density	~25,000+	3.4	2,640,000 (20.3%)	Day 48

Vaccination Scenarios and Epidemic Suppression

There were five simulation vaccination coverage conditions: 20, 35 (base), 50, 70, and 90 percent of the total population have been vaccinated at the start of the model, and the vaccination rate of n is applied to the susceptible compartment. Initial V(0) was adjusted to cover the thresholds.

At 20 percent coverage, the epidemic continued with slight suppression with a peak of 2,280,000 people (17.5 percent) and a total of 7,920,000 (60.9 percent) people infected in 180 days. At the 35% coverage, we obtained the following results as in Section 4.2. The peak of the epidemic with 50 percent coverage was lowered to 1,120,000 (8.6) and it was experienced at day

89, with total infections decreased to 5,240,000 (40.3) or a 22.7 percent decrease against the baseline. When coverage was 70 percent, which is close to the WHO-suggested coverage to a variety of respiratory diseases, the epidemic peak was catastrophic, at 310,000 (2.4 percent) on day 145, and the cumulative infections were 1,980,000 (15.2 percent). This is an example of a 70.8 percent decrease in cumulative infections relative to the baseline scenario, which depicts the strong nonlinear impact of the approach to the herd immunity threshold.

The model achieved near-complete epidemic control at covering 90 per cent with a peak number of 42, 000 people simultaneously infected and an overall number of less than 380, 000 people infected during 180 days (2.9%). At this level of coverage, the effective reproduction number was never greater than 1.0 and did not allow epidemic amplification. These results are in line with the theoretical herd immunity computations (Fine et al., 2011) and the empirical data on the high-income nations that reached high levels of vaccination coverage in relation to COVID-19 (Kraemer et al., 2020).

Table 3: Epidemic Results under different scenarios of covering vaccination

Coverage (%)	Peak Infections	Peak Day	Cumulative Infections	Reduction vs. Baseline
20%	2,280,000 (17.5%)	Day 52	7,920,000 (60.9%)	−16.8%
35% (Baseline)	1,842,000 (14.2%)	Day 67	6,780,000 (52.2%)	Reference
50%	1,120,000 (8.6%)	Day 89	5,240,000 (40.3%)	−22.7%
70%	310,000 (2.4%)	Day 145	1,980,000 (15.2%)	−70.8%
90%	42,000 (0.3%)	Day 180+	380,000 (2.9%)	−94.4%

Scenarios of Human Mobility Restriction

Three mobility scenarios were evaluated by modifying the mobility scaling factor m(t) uniformly across the simulation period: (1) full mobility (m = 1.0, baseline), (2) moderate restriction (m = 0.75, representing a 25% reduction from baseline), and (3) substantial restriction (m = 0.50, representing a 50% reduction consistent with observed lockdown-induced mobility reductions documented in Google Community Mobility Reports [2022] for Pakistan during COVID-19 restrictions).

At baseline mobility, there were the results as in Section 4.2. With a moderate restriction (m = 0.75), R0 decreased to 2.1 compared to 2.8 and the epidemic peak was decreased to 1260,000 (9.7) at the 88th day. The cumulative infections more than 180 days decreased to 5,560,000 (42.8%), which is smaller by 18.0% than with complete mobility. With significant pressure (m = 0.50), R0 additionally decreased to 1.4 and the epidemic peak decreased to 680, 000 (5.2) on day 118. Cumulative infections decreased to 4,180,000 (32.2%), which is 38.3% smaller than the baseline- consistent with the estimates of Chinazzi et al. (2020) and Kraemer et al. (2020) of mobility-reduction interventions.

Notably, even with mobility restrictions, but without simultaneous vaccination, it was also shown in the simulations that the measure was inadequate to restrain R[?] to below 1.0 at the initial vaccination coverage of 35%. At m = 0.50, the effective reproduction number was still greater than one on day 92 meaning that the epidemic was growing persistently over a very long duration. The fact highlights the complementarity of mobility restriction and the concept of vaccination as intervention measures and limitations of mobility-only interventions in the population with sub-optimal vaccination coverage.

Sensitivity Analysis Results

Normalized sensitivity indices were computed for the five primary model parameters: β, σ, γ, ν, and m. The results are presented in Table 4. The transmission rate β exhibited the highest positive sensitivity index (Y_β = +1.0), indicating that a 1% increase in β produces a proportional 1% increase in R_0—a consequence of the linear dependence of R_0 on β in the model formulation. The mobility factor m demonstrated an identical pattern (Y_m = +1.0), consistent with its multiplicative role in the transmission term. The recovery rate γ showed a strong negative sensitivity index (Y_γ = −0.96), reflecting the inverse dependence of R_0 on the infectious period. The vaccination rate ν exhibited a moderate negative index (Y_ν = −0.44), indicating that accelerating the vaccination process yields meaningful reductions in R_0 but is less immediately influential than transmission or recovery parameters. The incubation rate σ showed negligible sensitivity (Y_σ ≈ 0.00), consistent with theoretical expectations for SEIR models in which σ affects epidemic timing but not the equilibrium value of R_0.

These rankings were proven by univariate perturbation analysis. A 20 percentage point growth of β resulted in a 22.4 percentage point growth of cumulative infections whereas a 20 percentage point growth of n decreased cumulative infections by 9.8 percentage points. The most significant parameter perturbation was a 20% decrease in m, resulting in a 18.6% decrease in cumulative infections, which indicates that mobility control is a potent epidemic response, especially during the inability to scale-up vaccination rapidly. Incubation rate σ perturbations caused positive but insignificant epidemic population shifts of ±4–7 days but no significant changes in cumulative infection numbers, which agrees with the sensitivity index results.

Table 4 Sensitivity Indices of R_0 (Normalized) with respect to Model parameters

Parameter	Description	Sensitivity Index	Perturbation Impact (±20%)
β (beta)	Transmission rate	+1.000	+22.4% / −18.9% infections
m(t)	Mobility factor	+1.000	+19.7% / −18.6% infections
γ (gamma)	Recovery rate	−0.960	+15.3% / −14.2% infections
ν (nu)	Vaccination rate	−0.440	+10.2% / −9.8% infections
σ (sigma)	Incubation rate	~0.000	Peak timing ±4–7 days only

Discussion

The epidemiological evidence provided by the simulation outcomes in this paper supports and adds on the available body of epidemiological research on factors that determine the spread of epidemic diseases in the overcrowded urban settings. The observation that R0 in high-density regions of Multan districts is 3.4 is consistent with the claim reported by Rocklov and Sjodin (2020) that urban density studies at granular geographic scales are able to identify as systematically underestimated transmission risks that are not as recognized by aggregate methods. Of particular policy interest is the dramatic nonlinearity in the relationship between vaccination coverage and the outcomes of epidemics: the almost fourfold difference in cumulative infections between 50 percent and 70 percent vaccination coverage demonstrates why the importance of the herd immunity threshold is so critical as an operational measure of population health, which is consistent with the theoretical framework described by Fine et al. (2011) and the empirical evidence discussed by Plotkin et al. (2018).

The interventions associated with the complementarity between vaccination and mobility restriction have been shown to be complementary, i.e. the mobility-only intervention, which has 50% of the reduction capacity, was not enough to mitigate R to less than 1.0... this has great implication on the pandemic preparedness planning in Pakistan. The findings indicate that the public health authorities in Multan need to seek coordinated multi-intervention approaches that will simultaneously intensify the process of vaccination and implement mobility control tools which will be adjusted to the density distribution of the concerned areas. These results of the sensitivity analysis support this message: b and m(t) are equally powerful in their effect on R0, implying that interventions based on social contact (e.g., physical distancing, capacity limits on venues) and interventions based on movement (e.g., travel restrictions, curfews) have an equal potential to control the epidemic and should be perceived as complementary measures instead of substitutes. The moderate negative sensitivity of the vaccination rate n, although less than both b and m, suggests that even a small improvement in the daily vaccination throughput will produce significant reduction in epidemic in the long term, and justifies investment in the vaccination infrastructure and logistic.

Conclusion

This paper designed and used a more elaborate SEIR mathematical model to mimic the dynamics of epidemic diseases in Multan, Pakistan, using population density, vaccination coverage, and human mobility as the key predictors of disease spread. The national and international secondary data were used as parameters to parameterize the model and implemented computationally in Python. The major conclusions were that population density significantly increases the basic reproduction number and shortens the duration of the epidemic peaks; vaccination coverage higher than 70 percent has dramatic and nonlinear effects in reducing the severity of epidemics; mobility restriction at a 50 percent level produces significant but insufficient epidemic control in the absence of concurrent vaccination; and rate of transmission and mobility factor is the most influential parameter in the deterministic effect of epidemics outcomes. All of these results make it clear, that no single intervention is adequate to control epidemics in a high-density and mobility-heavy city such as Multan, and that only multi-component approaches can be effective.

Recommendations

Considering the results of this research, the following recommendations are proposed to the authorities of the local health and policymakers in Multan and similar cities in Pakistan. The primary recommendation is first, vaccination coverage must be the initial epidemic control tool with the target of at least 70 percent of the population coverage before the epidemic occurs or during the initial phases of the epidemic to take advantage of the nonlinear suppression effect shown by the model. Second, the district-level density mapping of the area must be included in the structure of public health planning to enable the application of interventions in the most densely populated areas of Multan, where the majority of the epidemic processes are most intense. Third, strategies of mobility management must be sustained as add-on response to vaccination, especially in times of vaccine shortage, and evidence-based fine-tuning of mobility controls can be used to ensure that the inter-district movement is reduced by at least half during the epidemic peaks. Fourth, mobility data collection in real time, using the available digital data streams, should be institutionalized in the Multan public health information system so that the epidemic model parameters could be updated dynamically. Fifth, the modeling framework that was constructed in the context of the present study needs to be further expanded by age-structured mixing matrices and healthcare capacity constraints, which are the most significant gaps of the present formulation. Sixth, the Pakistani academic and public health institutions should invest in epidemic modeling capacity in order to facilitate timely and context-sensitive epidemic analysis that informed international reactions to COVID-19 but was largely lacking in the Pakistani reaction.

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